Assessment of the curvature and slope for the VC-Ogee relationship allowing for 3D flow

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1. Introduction and background

The van Vuuren and Coetzee (VC) Ogee relationship was introduced at ICOLD 2016 (van Vuuren & Coetzee, 2016) and is the only Ogee profile relationship that compensates for the effects of 3-dimensional flow that results from the upstream approach channel. This VC-Ogee relationship is distinguishable due to its derivation from both physical and numerical modelling of prominent 3-dimensional flow in the approach channel. The derived relationship uses four parameters to adjust the curvature and slope of the Ogee profile, depending on the upstream approach channel layout conditions. When compared to existing relationships (accounting only for 2-dimensional flow), the VC-Ogee relationship improves the crest pressure distribution resulting in reduced negative pressures for flows exceeding the design head. Furthermore, the VC-Ogee relationship has a gradual slope, with no abrupt change in the curvature, thereby reducing the possibility of flow breakaway that may contribute to cavitation damage of the structure.

The widely used WES relationship is one of the most studied hydraulic relationships in dam engineering (Savage & Johnson, 2001). In the past, various attempts have been made to accurately describe the shape of the lower nappe of water flowing across an aerated, sharp-crested weir, known as the Ogee profile. The Ogee spillway can be divided into three regions: the upstream quadrant, the downstream quadrant and the rear slope (Fig. 1). Due to its high discharge efficiency, the nappe-shaped profile is used for most spillway control crests (Khatsuria, 2005).

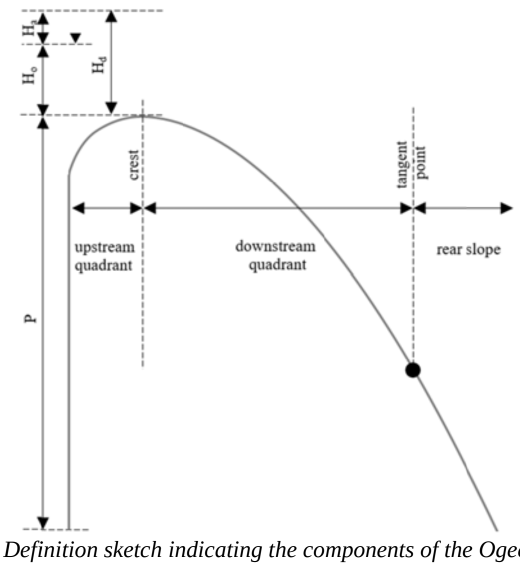


Fig 1. Definition sketch indicating the components of the Ogee profile

Existing Ogee profile relationships were formulated from 2-dimensional hydraulic assessments. Recent studies that included the variation of the upstream geometry in the approach channel, have indicated that the existing 2-dimensional Ogee relationships may be insufficient when 3-dimensional flow is present (van Vuuren, *et al*., 2011). The effect of contraction, oblique and asymmetrical approach channels and curvature of the dam wall contribute to 3-dimensional flow.

An accurate Ogee profile improves the hydraulic efficiency of the spillway structure and minimizes the possible occurrence of sub-atmospheric pressure that may contribute to cavitation formation on the spillway’s surface. When considering an Ogee spillway, which was designed using a 2-dimensional flow relationship, with an asymmetrical and/or oblique approach channel, 3-dimensional flow across the Ogee spillway will be present. For flows exceeding the 2-dimensional design head (H_d), sub-atmospheric pressures will be present at the surface of the spillway, this may contribute to more detrimental effects such as separation of the lower nappe from the spillway's surface and/or cavitation of the spillway.

Montes (1992) has indicated that numerous attempts have been made by researchers to improve the design of the Ogee profile. Pre-1950 the evaluation of the Ogee profile was done by means of physical modelling. Thereafter, potential flow solutions of the sharp crested weir flow had been obtained by numerical calculations (Mc Nawn, Hsu and Yih (1953), Strelkoff (1964)). Hay and Markland (1958) have even attempted solving the profile by means of electric analogy.

More recently, continuous Ogee spillway profiles have been developed. This is after several researchers have recognized that any sudden variation in the slope and/or curvature of the profile near the crest should be avoided Montes (1992). The first continuous Ogee profile was proposed by Knapp (1960) followed by Hager (1987), Montes (1992) and most recently by van Vuuren and Coetzee, (2016). The most recent, also known as the VC-Ogee relationship, is the first Ogee profile to include the effects of 3-dimensional flow from the upstream approach channel. The formulation of the VC-Ogee relationship is unlike any of its predecessors that have only considered 2-dimensional flow in an unconstructed, symmetrical flume. The progress of development of the Ogee profile for spillway design is reflected in Fig. 2.

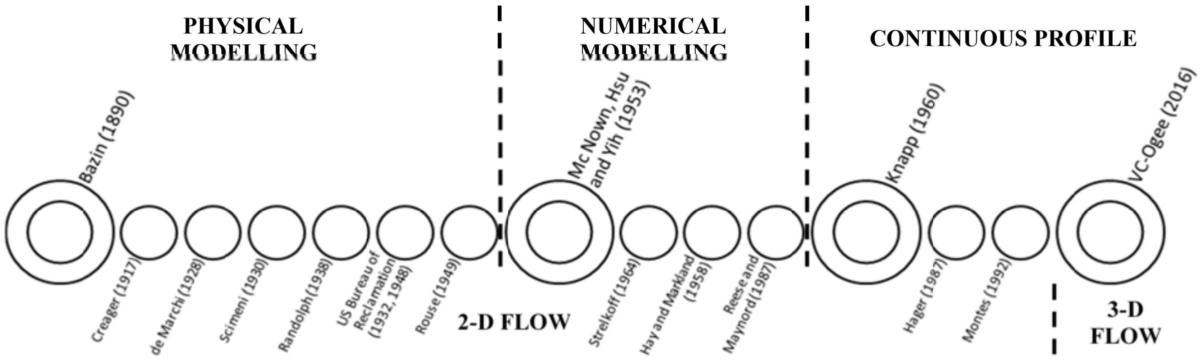


Fig 2. Historical development of the Ogee profile

All these Ogee profiles have small but significant differences, particularly near the crest of the weir where the potential flow profile departs tangentially from the weir wall. According to Montes (1992) most of the Ogee profiles have a departure angle of about 27°, due to unavoidable boundary layer effects caused by the interception of horizontal flow over the crest and upward flow from the weir wall. A summary of the most renowned profiles are reflected in Fig 3 (Coetzee, 2013):

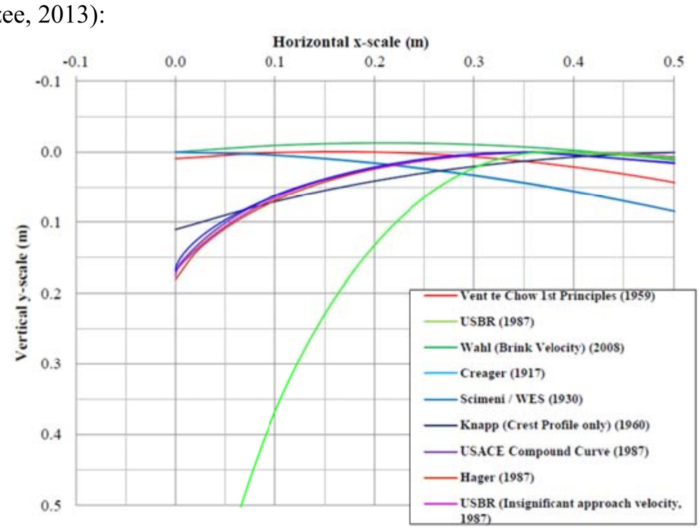


Fig 3. Comparison of the crest region of the most renown 2-dimensional Ogee profiles (Coetzee, 2013)

In the regime of 2-dimensional flow, dams that were constructed with the Bazin profile revealed that as long as the head in the dam did not exceed the head on the sharp crested weir that originated the profile, the pressures on the face of the dam remained positive, with the exception of a zone near the crest of the dam where negative pressures were commonly observed as shown in Fig. 4. A small variation in the crest geometry significantly alters the surface pressure distribution on the crest of the spillway (Grzywienski, 1951).

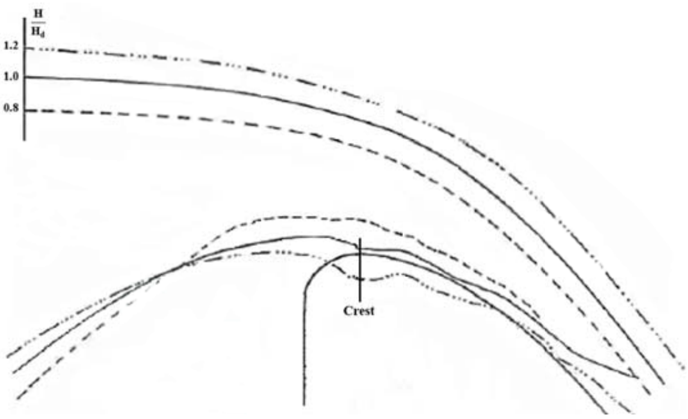


Fig 4. Pressure variations at the crest region due to small changes in the Ogee profile geometry (Grzywienski, 1951)

The presence of 3-dimensional flow results in an increased area of this negative pressure region. With increased discharge, the negative pressure region causes the formation of vapour pressure voids, which are rushed downstream by the sheer velocity on the spillway to a region of higher pressure. This is generally on the way to the end of the spillway. The vapour pressure voids are then imploded, possibly near the surface of the spillway which could result in significant cavitation damage of the concrete. A recent case study at Nagarjuna Sagar Dam in India, showed cavitation damage causing the concrete of the spillway to be eroded up to 1.8 m deep during a series of floods in 2009. A model study of this dam indicated that negative pressures were experienced on the crest of the Ogee spillway for floods of only 25% of the original design head (H_d) (Rajasekhar, et al., 2014).

In this paper, the curvature and slope of the VC-Ogee relationship are compared with some of the most renowned Ogee profiles used historically for dam spillway designs (USACE, 1970 and USBR, 1987, known as the WES-Ogee Profiles as well as Hager's, 1987, continuous profile). Furthermore, the improvement of the crest pressure distribution for the VC-Ogee relationship is discussed.

2. Ogee profiles used for comparison

A comparison of the various 2-dimensional Ogee profile relationships with the VC-Ogee relationship is provided in the successive sections.

2.1 U.S. Army Corps of Engineers (USACE, 1970 revised 1987)

The U.S. Army Corps of Engineers (USACE, 1970, revised 1987) suggested a compound circular curve to describe the upstream quadrant of the Ogee profile. Similarly, the upstream quadrant defined by the USBR (1987) resulted in a surface discontinuity at the vertical spillway face. Model studies at the U.S. Army Waterways Experiment Station indicated that the incorporation of a small arc with radius = $0.04H_d$ actually improved the pressure conditions, reduced possible cavitation of the spillway crest and increased the discharge coefficients for heads exceeding the design head (USACE (a), 1987). The coordinates for the origins of curvature and transition points of the improved design is schematically indicated in Fig. 5 together with the numerical values of the origins.

The downstream quadrant of the Ogee curve is defined by a power function (Equation 1) similar to the description used by the USBR (1987). The USACE (1987) define the K and n coefficients in the equation as 0.5 and 1.85, respectively.

$$\frac{y}{H_d} = -K \cdot \left(\frac{x}{H_d}\right)^n \qquad \textbf{Equation 1}$$

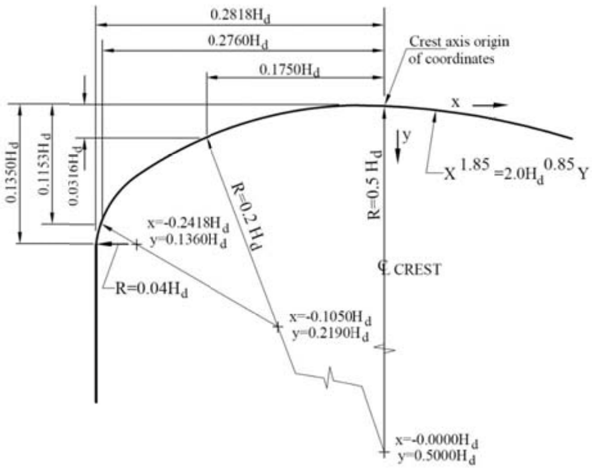


Fig 5. The coordinates for the upstream quadrant of the Ogee curve as defined by the USACE (1987)

2.2 U.S. Bureau of Reclamation (USBR, 1987)

Research conducted by the United States Bureau of Reclamation (USBR, 1987) has resulted in the approximation of the Ogee profile by means of a series of compound circular curves. The conclusion of this research resulted in the formation of a relationship that numerically described the shape of the Ogee curve with relation to the design head of the system. According to the USBR (1987), the shape of the Ogee profile dependents on the following factors:

- Design head;
- Upstream wall face inclination; and
- Pool depth, which in turn influences the approach velocity.

The relationship for the curve derived by the USBR consists of two quadrants: the first being the portion upstream from the apex of the Ogee profile, and the second being downstream from the apex. The upstream quadrant can be defined in two ways, namely as a single curve in combination with a tangent, or as a compound circular curve. It is, however, mostly defined using the latter. The downstream portion can also be described as a power function or as a compound circular curve.

The approximate Ogee profile for a crest with a vertical upstream face and negligible approach velocity can be estimated by the compound circular curve configuration. This comprises of two arcs of differing radii and points of origin for the upstream quadrant of the spillwayError! Reference source not found.. The downstream quadrant of the profile is estimated by five circular curves of differing radii and points of origin. Vischer & Hager (1999), indicated that the ratio of the velocity head to the design head is negligible, provided that the head over the weir is greater than 100 mm and smaller than half of the pool depth.

2.3 Hager, 1987

Vischer & Hager (1999) noted that the Ogee profile approximated by the USBR (1987) and USACE (1987) was made up of multiple compound circular curves. This is disadvantageous as there are sudden discontinuities on the Ogee profile that occur at each of the transition points where the arcs intersect, resulting in cavitation problems at the discontinuities during high flow.

Hager (1987), approximated the Ogee curve by an alternative smooth curve determined by transposing the co-ordinates given by the USACE (1987). Hager (1987) has transferred the origin of the co-ordinate system used by the USACE (1987) located at the highest point of the Ogee's crest to the minimum horizontal distance value. The Ogee curve that was suggested by Hager (1987) can be mathematically calculated by means of Equation 2.

$$Z^*=-X^*\ln X^*, \text{ for } X^*>-0.2818 \qquad \textbf{Equation 2}$$

X^* and Z^* are the transformed coordinates based on the three arc compound circular curve of USACE (1987) as depicted in Equation 3 and 4 with x and z being the Cartesian co-ordinate of the Ogee profile.

$$X^{*}=1.3055\cdot\left(\frac{x}{H_d}+0.2818\right) \qquad \textbf{Equation 3}$$

$$Z^{*}=2.7050\cdot\left(\frac{z}{H_d}+0.1360\right) \qquad \textbf{Equation 4}$$

2.4 VC-Ogee Relationship

The VC-Ogee relationship was developed by considering the variation of the lower nappe for flow over a sharp-crested weir as a result of 3-dimensional flow that was present from the upstream approach channel. This relationship applies a uniform, smooth sloped numerical function for determining the Ogee profile of the spillway, which may be expressed in Macaulay notation. (van Vuuren & Coetzee, 2016).

The VC-Ogee relationship in the Macaulay notation is reflected in Equation 5:

$$\text{VC-Ogee}=\begin{cases} z=-H_e\cdot B\left(\left(\frac{x}{H_e\cdot A}+\frac{1}{e}\right)\cdot\ln\left(\frac{x}{H_e\cdot A}+\frac{1}{e}\right)+\frac{1}{e}\right) & \text{for the upstream quadrant} \\ z=\frac{x^{C}}{D\cdot H_e^{(C-1)}} & \text{for the downstream quadrant} \end{cases}$$

Equation 5

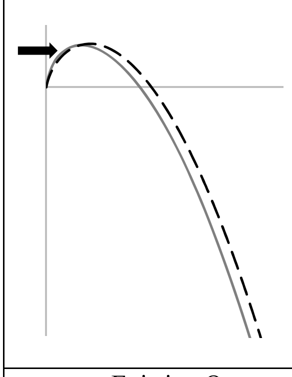
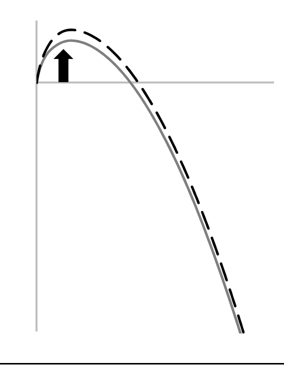
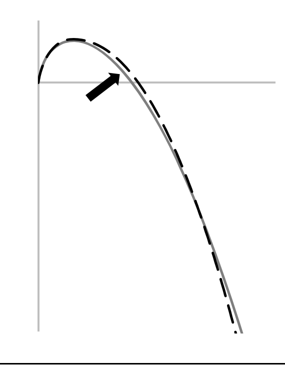
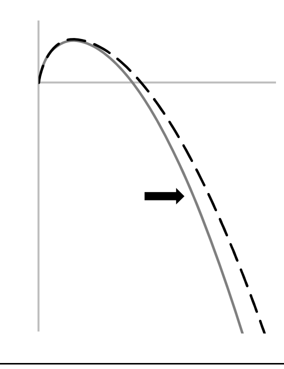
From Equation 5, the log-normal-term (ln-term) $\left(\frac{x}{H_e\cdot A}+\frac{1}{e}\right)>0$ defines the vertical asymptote. Depending on the slope of the upstream wall, this term can be used to determine a tangent point on the upstream side of the Ogee profile ensuring a smoother transfer of the curvature from the upstream wall to the Ogee profile (van Vuuren & Coetzee, 2016).

The downstream quadrant of the Ogee profile was defined by using a modified version of the WES power function (Equation 6). This corresponds to the approach followed by the USBR (1987) and USACE (1970). The upstream and downstream quadrants intercepts at $\frac{dz}{dx}=0$. This allows for a smooth uniform transfer of the curvature without any discontinuities.

$$z=\frac{x^{C}}{D\cdot H_e^{(C-1)}} \qquad \textbf{Equation 6}$$

The four parameters A, B, C and D which denotes the horizontal translation (A), vertical elongation (B), curvature (bulging effect) (C) and magnitude (D) were incorporated to alter (increased and/or decreased) the Ogee profile. With these alterations 3-dimensional flow conditions may be incorporated, alleviating sub-atmospheric pressures (See Table 1). The numerical values of these parameters were given by van Vuuren and Coetzee (2016).

Table 1. Exaggerated illustration of the influence of the different parameters (van Vuuren & Coetzee, 2016)

Parameter A: **Horizontal translation of the upstream quadrant**	**Parameter B:** **Vertical displacement of the upstream quadrant**	**Parameter C:** **Bulging of the downstream quadrant**	**Parameter D:** **Increase the extent of the downstream quadrant**
			
——— Existing Ogee profile considering 2-dimensional flow -------- Influence of VC-Ogee parameters to incorporate the effect of 3-dimensional flow			

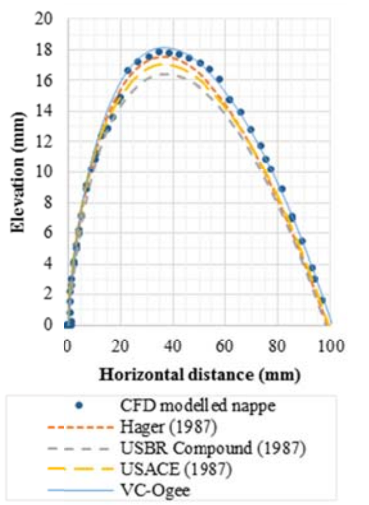
3. Results and discussion

A comparison between four different Ogee profiles were made for three typical design scenarios. These design scenarios included:

- Layout A: a uniform approach channel with flow perpendicular onto the dam wall. The spillway reduced the effective width of the approach channel;
- Layout B: similar to layout A, but included a curved dam wall, instead of the straight dam wall; and
- Layout C: similar to layout A, with the exception that the approach channel was at an oblique angle of 12.47 degrees.

The different Ogee profiles were plotted in XZ-coordinates and compared with the lower nappe obtained from the numerical model by van Vuuren and Coetzee (2016). The comparison of the Ogee profile's crest region for each of these layouts are depicted in Table 2.

Table 2. XZ-plot of the Ogee profiles for the different layouts

<table>
<tr><th>Layout A</th><th>Layout B</th><th>Layout C</th></tr>
<tr>
<td>
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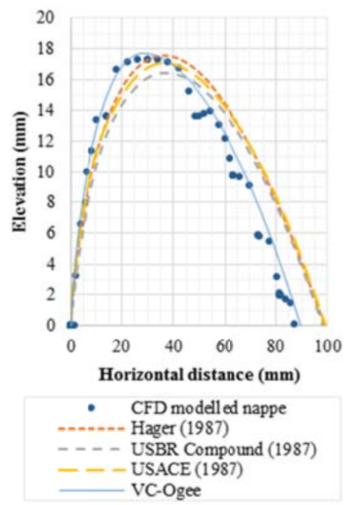
<td>
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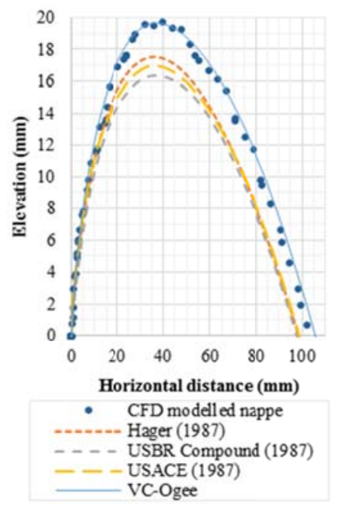
<td>
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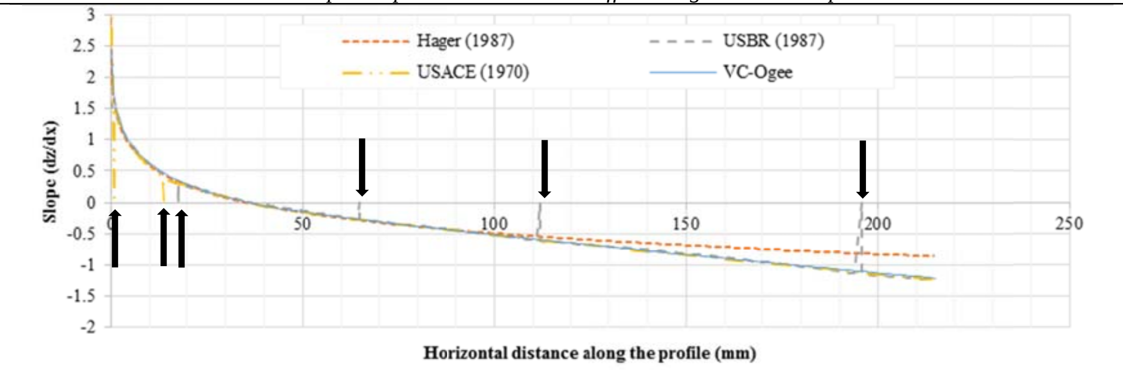
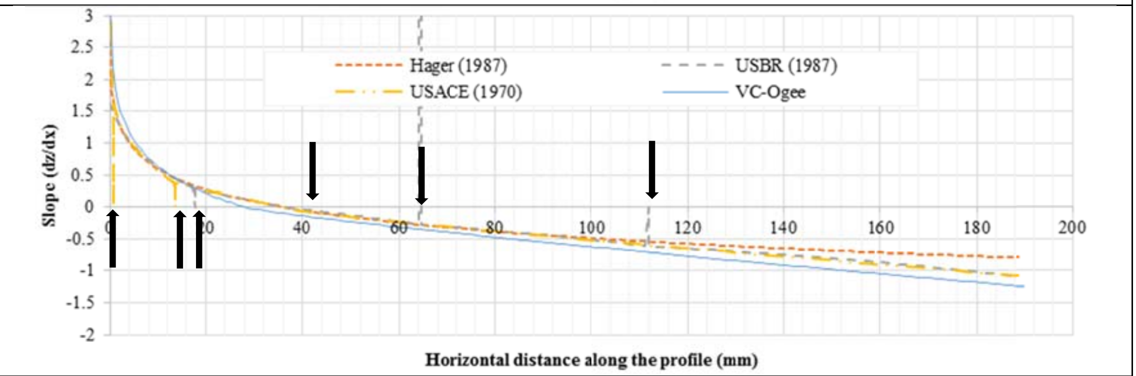
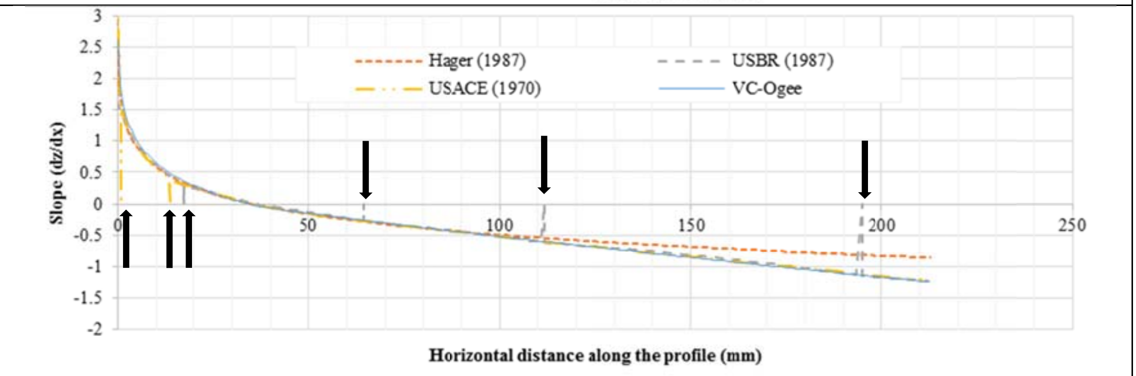
The profile plots indicated the presence of 3-dimensional flow in all scenarios, with B and C being the most significant. In all cases there was a definite discrepancy between the modelled lower nappe and the 2-dimensional Ogee profiles. The 2-dimensional Ogee profiles all fell within a 1 mm envelope of each other, which indicates the reliability in the methods that were historically used to develop these relationships. However, the existing 2-dimensional flow relationships were insufficient in approximating the lower nappe and tend to underestimate its geometry. Although these variations may in some cases be marginal, in scenarios B and C they were significant. The VC-Ogee relationship represented the lower nappe of the Ogee profile accurately in all three these layouts.

Furthermore, to ensure the effective and efficient design of Ogee spillways, a gradual slope and curvature change of the Ogee profile must be maintained. The VC-Ogee relationship has a smooth and gradual slope change similar to the USACE (1970; 1987) and USBR (1987) relationships with the added advantage of having no discontinuities. The VC-Ogee relationship also improves upon the curvature of the downstream quadrant as suggested by Hager (1987). The comparative slopes between the different relationships are given in Table 3. The discontinuities of the relationships being derived by compound arcs, are reflected as vertical asymptotes indicated with the black arrows. An upward arrow reflects the discontinuity on the upstream quadrant, while a downward arrow reflects the discontinuity on the downstream quadrant (Table 3).

The VC-Ogee relationship eliminates the discontinuities that were present in the existing 2-dimensional relationships. Breakaway of the lower nappe is also mitigated with the VC-Ogee profile while the formation of negative pressures at increased heads are reduced. The crest pressure distribution was numerically modelled with computational fluid dynamics. This pressure distribution on the surface of the spillway for Layout C, at an increased head of 50% above the design head is shown in Fig. 7. The results indicated that the maximum negative surface pressure is reduced by 10%. These results further accentuate the increased safety of the VC-Ogee relationship and establish the validity in considering 3-dimensional flow when designing an Ogee spillway.

Table 3. Slope comparison between the different Ogee relationships

Layout A	
Layout B	
Layout C	

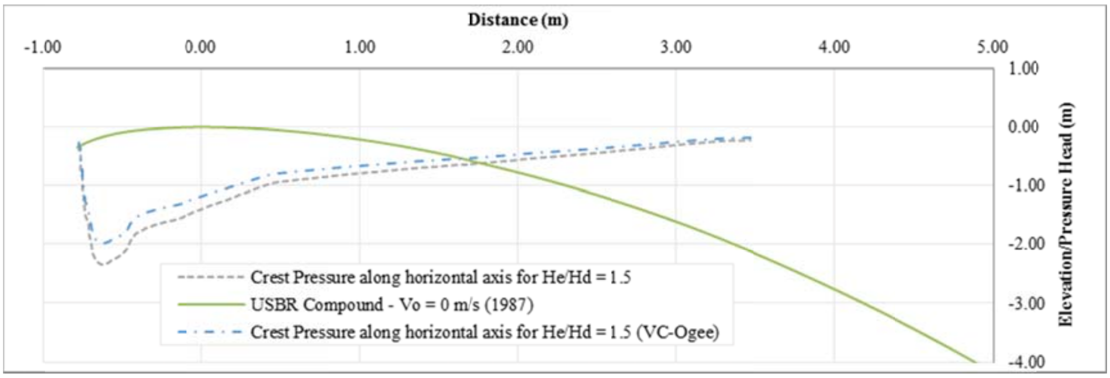


Fig 7. Improved surface pressure distribution for the VC-Ogee Relationship

4. Conclusion

The VC-Ogee relationship has a smooth uniform Ogee profile that incorporates the effect of 3-dimensional flow. It has no abrupt changes in curvature or slope, thereby reducing the possibility of breakaway or cavitation damage that may develop.

Furthermore, the VC-Ogee relationship has proved to reduce the negative pressure distribution on the crest of the spillway with heads above the design head (H_d). In order to improve dam safety and to limit spillway damage during extreme floods, the VC-Ogee relationship should be considered when designing Ogee spillways.

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