Application of the First Order Reliability Method to Planar-Type Failure in a Hard-Rock Slope to Assess the Probability of Failure

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ABSTRACT
The First Order Reliability Method (FORM) is a semi-probabilistic reliability analysis method devised to evaluate the reliability of a system. The stability of the system must be a function of two or more probabilistic variables that have a mean, and standard deviation. The correlation between the probabilistic variables must be determined, which can be difficult, as will be show, independent variables can be functions of independent variables. The FORM considers any deterministic variables also included in the stability function as having a mean equal to their value, and a standard deviation of zero.

The equations for quantifying the stability of a planar-type failure mechanism in a hard-rock slope are available in close-form and can be easily implemented in a spreadsheet, and checked against a commercially available limit equilibrium method software program. The author uses a worked example to show the application of FORM to a hard-rock slope susceptible to the planar-type failure mechanism. The author provides commentary on the application of the FORM in engineering practice given that the user will find it difficult to define or measure the correlation factor(s) and reduction factors.

RÉSUMÉ
Le premier fiabilité méthode (formulaire de commande) est une méthode d'analyse de fiabilité probabiliste semi conçue pour évaluer la fiabilité d'un système. La stabilité du système doit être une fonction de deux ou plusieurs variables probabilistes qui ont une moyenne et écart-type. La corrélation entre les variables probabilistes doit être déterminée, qui peut être difficile, comme se montrera, variables indépendantes peuvent être une fonction des variables indépendantes. La forme considère toutes les variables déterministes compris également la fonction de stabilité comme ayant une moyenne égale à leur valeur et un écart-type de zéro.

Les équations pour quantifier la stabilité d'un mécanisme d'échec plane-type dans une pente de roche dure sont disponibles sous forme de clôture peuvent être facilement mis en œuvre dans une feuille de calcul et vérifiées contre un logiciel de méthode équilibre limite disponible dans le commerce programme. L'auteur utilise un exemple pour montrer l'application de formulaire à une pente de roche dure susceptibles d'être le mécanisme de rupture plane-type. L'auteur fournit le commentaire relatif à l'application de la forme à la pratique de l'ingénieur étant donné que l'utilisateur sera difficile de définir ou de mesurer les facteurs de corrélation et les facteurs de réduction.

Original

1 INTRODUCTION

The assessment of risk in geotechnical engineering can be daunting if not near impossible owing to the inherent heterogeneous and anisotropic nature of geomaterials. It can also be cost prohibitive at times to collect sufficient data to make a reliable assessment of the mechanical, spatial, and temporal variation of materials at a given site. However, recent events have occurred that have shined a light on the fact that geotechnical engineers should be doing more to quantify risk (Guthrie 2017).

Despite the difficulty in assessing risk, methods have been developed that can be applied to common geotechnical engineering problems to assess the probability of failure, which is typically the missing quantity when assessing risk, where is risk is defined as the product of the consequence of failure and the probability of failure occurring (Fenton and Griffiths 2008, Low 2008) as long as there is a closed form solution for the problem.

This paper presents the implementation of the first order reliability method (FORM) in a spreadsheet to assess the probability of failure of a planar-type rock slope failure in a rock mass. This method relies on the collection of data that can be fitted to a statistical distribution such that a mean and standard deviation can be determined for a given independent random variable. The inclusion of deterministic variables is allowed, so long as there are two or more independent random variables for the calculation of a correlation coefficient.

The use of a spreadsheet is ideal for FORM when a closed form solution is available for a given stability problem. For a planar-type failure on a hard rock mass, the commonly used Microsoft® Excel software has all the necessary built in functions required to complete the analyses, with the exception of the truncated exponential distribution (Olive 2008).

2 PARAMETERS

2.1 Definition

The parameters typically collected during a geological mapping program or borehole drilling program (ideally orientated core when assessing the stability of a rock slope) are used in this FORM analysis. The measured values from the field (i.e. those variables that define the structural domain of the rock mass) become known as independent random variables in the analysis. The problem is illustrated in Figure 1 where the measured independent random variables for the failure block height from toe to crest, H_f, the dip of the failure block face, ψ_f, the dip of the failure plane, ψ_p, the distance from the crest of the block to the tension crack, b, with the calculated independent random variables for the depth of the tension crack, z_{TC}, the horizontal water pressure acting on the tension crack, U, and the normal water pressure acting on the failure plane, V. The length of the failure plane, L, over which V acts is not indicated for clarity.

It should be noted that z_{TC} was chosen as vertical to simplify the problem used in this study. In real world applications the location of z_{TC} would be defined by the spacing of the back release joint set, if present. The angle of z_{TC} would be set by the dip of the back release joint set, again, if present. A further simplification was made by assuming the crest of the block was horizontal, whereas in real world applications this would often not be the case unless intermediate benches were present in a rock cut slope for instance.

The measured independent random variables are listed in Table 1, with the corresponding distribution used in this study to define the mean (μ), standard deviation (σ), and variance (σ^2) of the variables. The mean is the first arithmetic moment of the normal distribution. The truncated exponential distribution requires the parameter λ, which is equal to the inverse of the mean. It should be noted that some researchers have defined λ as the mean of the exponential distribution (Olive 2008).

The mean is also defined as the expectant value, $E[X]$ and the variance, $Var[X]$. Two additional terms from the distributions are needed for this analysis, which are the second and third arithmetic moments, $E[X^2]$ and $E[X^3]$, respectively, where X is an independent random variable or function of one or more independent random variables.

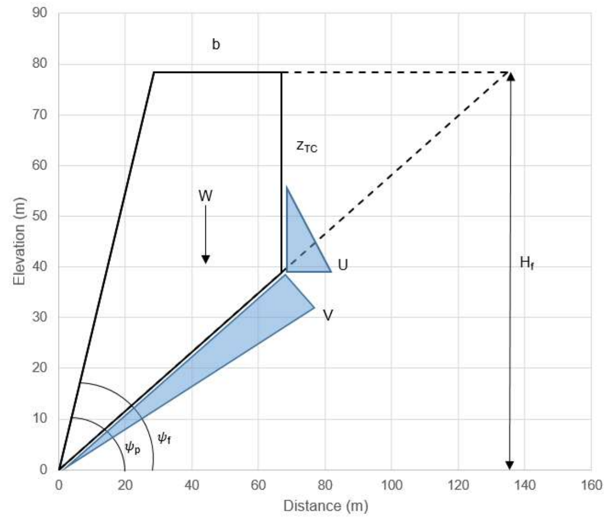

Figure 1. Illustration of a block susceptible to planar-type failure with measured and calculated independent random variables indicated.

Table 1. Measured Independent Variables and their Fitted Assumed Distributions

Parameter	Distribution[1]
ψ_p (°)	Log-Normal
ψ_f (°)	Log-Normal
H_f (m)	Truncated Exponential[2]
b (m)	Log-Normal
γ_r (MN/m^3)	Log-Normal
z_w (m)	Truncated Exponential

[1]mean and standard deviations reported below in Table 2
[2]the proof for the truncated exponential distribution is provided by (Olive 2008)

Two functions are said to be dependent if they both involve the same independent random variable (Fenton and Griffiths 2008).

2.2 Independence

Independence can be defined as a variables ability to occur independent of the occurrence of another variable (Fenton and Griffiths 2008). For instance, the joint set forming the face of the slope is independent of the joint set forming the planar sliding surface. Another example would the independence of water collecting in a tension crack from the angle defining dip of the crest at the top of the slope.

Independence is not an indication of correlation or lack thereof, which is discussed below (Fenton and Griffiths 2008). Dependency of one variable on another clearly invokes correlation between the two. For instance, if W and Z are both functions of X, then W and Z are clearly dependent variables and will be correlated.

2.3 Correlation

Correlation can be defined as the affect one random variable on another, or the direct measure of the degree of linear dependence between two variables (Fenton and Griffiths 2008). Linear correlation is best defined by the parameterized correlation coefficient, given by Equation 1 (Triola 1999, Fenton and Griffiths 2008).

$$\rho_{XY} = \frac{Cov[X,Y]}{\sqrt{Var[X]}\sqrt{Var[Y]}} \quad [1]$$

Where $Cov[X,Y]$ is defined as the covariance of two variables and is defined by Equation 2, and the square root of variances of X and Y or simply their standard deviations.

$$Cov[X,Y] = E[XY] - E[X]E[Y] = E[XY] - \mu_X\mu_Y \quad [2]$$

Where $E[X]$ is the expectant value of X which is its mean. The $E[XY]$ is the product of the two expectant values. When the $f_X(X)$ is multiplied by the $f_Y(X,W,Z)$, then $E[XY]$ would be $E[X^2WZ]$ requiring the second arithmetic moment of X. For instance, the height of a block susceptible to planar sliding, H_f, will have a covariance with the total length of a sliding surface due to the definition of A, as shown in Equation 2.

$$\begin{aligned} Cov[H_f, L] &= E[H_f L] - E[H_f]E[L] = E[H_f L] - \mu_{H_L}\mu_L \\ &= E[H_f \times (H_f - z_{TC}) \times csc(\psi_p)] - \mu_{H_L}\mu_L \\ &= E[H_f^2 csc(\psi_p) - H_f z_{tc} csc(\psi_p)] - \mu_{H_L}\mu_L \end{aligned} \quad [3]$$

Equation 3 shows how multiplying through the function for L results in the product of H_f^2, which is the second arithmetic moment for H_f, when inside the square brackets of the expectant value.

If $f_Y(Y)$ was expressed as $f_Y(X^2,W,Z)$, then the third arithmetic moment of X would be required and Equation 1 would no longer be valid, as the correlation would not be linear (Triola 1999). An example of a higher order function is the covariance of the height of water in a tension crack, z_w, and the vertical water pressure, V, shown in Equation 4.

$$\begin{aligned} Cov[z_w, V] &= E[z_w V] - E[z_w]E[V] = E[z_w V] - \mu_{z_w}\mu_V \\ &= E[z_w \times (0.5 \times \gamma_w \times z_w^2)] - \mu_{z_w}\mu_V \\ &= E[z_w^3 \times 0.5 \times \gamma_w] - \mu_{z_w}\mu_V \end{aligned} \quad [4]$$

A proof for the linear correlation coefficient implies that its value is defined over the range $-1 \le \rho \le 1$ (Fenton and Griffiths 2008). In order to have all correlation within the same range, a non-parametrized correlation is required for correlation, which would be the Spearman ranking correlation (Triola 1999). The author used the Spearman ranking correlation for all non-linear correlations, which can be identified by equations with variables raised to some power, or some other higher order function. The Spearman correlation is given by Equation 5.

$$\rho_s = 1 - \frac{6\sum d^2}{n(n^2-1)} \quad [5]$$

Whereρ_s is the Spearman correlation, $\sum d^2$ is the sum of the squared ranking numbers for each of the data points, and n is the number of variables being considered. The number of data points for each variable must be the same.

The correlation matrix for the FORM (discussed in further detail below) is then the matrix of the correlation coefficients for each of the independent random variables. This is easily defined by setting the sequence and rows of the matrix in the same order of variables and inputting the corresponding coefficients, with the diagonal of the matrix being a series of ones since the correlation coefficient of for a single variable would be the variance of that variable divided by the square of its standard deviation $(\sigma_X^2 = Var[X])$.

2.4 Random Variable vs. Deterministic Variables

Random variables, by definition, given enough data, can be fitted to distribution and have a defined mean and standard deviation. Deterministic values are constant, meaning their value is their mean and they do not vary. Their inclusion in probabilistic analysis using FORM results in no correlation and they can be discounted when creating the correlation matrix (see below).

Examples of deterministic variables the unit weight of water, the gravitational constant, and any measured value that does not have a defined standard deviation (i.e. the tensile strength of rock bolts – not provided by the manufacturers).

The analysis can be simplified when some of the measured independent random variables are defined as constants. Examples in engineering literature using FORM to assess the probability of failure of a rock slope have done this (i.e. Low 2008). The authors did this to simplify the analysis so as to focus in on the area of research. This paper is meant to provide a meaningful and instructive method to assess the probability of failure. The next section will discuss the values of the independent random variables used in this study, as well as where the author used deterministic values to provide an example where an engineer would normally assume imminent failure, but the probabilistic assessment provides clarity as to the actual likelihood of failure.

2.5 Values

Table 2 provides a summary of the values of the independent variables used in this study. Based on previous experience of the author, the geological mapping data was taken as being log-normally distributed. A random number generator in the spreadsheet was used to generate the random variables.

Table 2. Summary of the Values of the Independent Random Variables used in this study.

Parameter	Mean[1]	Standard Deviation[1]
ψ_p (°)	30.2	8.3
ψ_f (°)	70	1.2
H_f (m)	78.3	78.3
b (m)	38.2	15.8

γ_r (MN/m^3)	0.0259	0.00121
z_w (m)	14.4	14.4

[1]normal mean and standard deviations are reported here based on distributions reported in Table 1

As described above, some deterministic variables were used to set the stability of the planar-type failure to 1.0. The deterministic variables are listed in Table 3. The author would like to point out that the variables listed in Table 3 were chosen to be deterministic owing to the complexity of determining the covariance between them and other variables. Despite the fact that the variables in Table 3 are measured values, the covariance they would have with the Barton and Bandis shear strength function is evident, but due to the inclusion of the logarithmic and trigonometric functions, the difficulty of calculating the covariance would be daunting.

Table 2. Summary of the Values of the Independent Deterministic Variables used in this study.

Parameter	Mean	Standard Deviation[1]
φ_r	10	0
JRC	7	0
JCS	19.5	0

[1]standard deviation of a deterministic variable is always zero

The first order approximation method was used to determine the mean and standard deviation of the calculated dependent variables, as listed in Table 3 (Fenton and Griffiths 2008). Where trigonometric functions are applied to angles in functions for the calculated dependent variables, the first order approximation method was used to calculate the mean and variance of the angles when used with a trigonometric function. This simplified the calculation of the covariances between the various variables. The rationale for doing this, was that the data points for the measured independent random variables are discrete and not continuous (i.e. they are measured individually and typically define a single feature, such as the dip of joint, versus the dip of an adjacent but slightly steeper joint). Furthermore, the calculation of a dependent variable is independent of whether an angle is measure in degrees, which requires a trigonometric function, or whether the angle is measured in gradients. All the author did was convert the measured angles from degrees to gradients.

Table 3. Summary of the Values of the Dependent Variables used in this study.

Parameter	Mean	Standard Deviation[1]
Depth of tension crack, z_{TC} (m)	39.5	44.1
Area of the failure plane, A (m^2/m)	77.3	118
Weight of the sliding block, W (MN/m)	72.9	111
Horizontal water pressure, U (kPa/m)	5.47	9.96
Normal water pressure, V (kPa/m)	1.02	1.02
Normal stress, σ_n (MPa/m)	0.733	2.21
Shear strength, τ (MPa/m)	0.266	0.801
Partial differential of shear strength to normal stress, $\partial\tau/\partial\sigma_n$ (ratio)	0.303	0.912
Instantaneous friction angle, φ_i (grad)	0.303	0.912
Instantaneous cohesion, c_i (MPa)	0.263	0.790

[1]standard deviation of a deterministic variable is always zero

The author also would like to point out that using the first order estimation method (Fenton and Griffiths 2008) may not provide the most accurate estimates of variance. Further work is required to assess the accuracy of first order method by comparing it other methods.

3 FORM

3.1 Equation

The FORM is a method that determines the shortest path to the failure surface or limit state surface, as shown in Figure 2, from the mean point of stability (Low 2008). That is, the factor of safety is defined using the mean of the variables and then the standard deviations are used to determine how far said point is from the limit state surface. That distance is then assessed to determine the likelihood of the data defining the mean actually being at or past the failure surface. The closet point on the limit state surface to the mean point is known as the design point, as shown in Figure 2 (Low 2008), which denotes the position of unity with respect to the working limit state (i.e. factor of safety equal to one).

FORM was developed by Hasofer and Lind based on their work studying the first-order second-moment method (FOSM) (Fenton and Griffiths 2008). The solution to FOSM follows the gradient at the mean point to the failure surface which is not a unique solution in that may not actually be the shortest distance between the mean point and failure surface. FORM is a better representation of the probability of occurrence, however it does assume a linear failure surface. In the case of a non-linear failure surface, multiple local minima may occur which could result in an under estimation of the probability of occurrence (Fenton and Griffiths 2008). Given the use of the Barton and Bandis shear strength criteria in this study, and the continued use of a linear approximation of the Mohr-Coulomb strength criteria that Barton and Bandis is based on, the author considers FORM to be a suitable method for at least preliminary design, if not detailed design.

FORM is defined by Equation 6.

$$\beta = \min_{M=0} \sqrt{\left(\frac{x-E[X]}{\sigma_X}\right)^T \boldsymbol{C}^{-1} \left(\frac{x-E[X]}{\sigma_X}\right)} \quad [6]$$

Where β is the reliability index (discussed further in Section 2.7), M is the limit state surface, $\boldsymbol{x}$ is the vector of number of standard deviations from the mean for each independent random variable, and $E[\boldsymbol{X}]$ is the vector of means for the variables, $\boldsymbol{C}$ is the inverse of the correlation matrix. The superscript T indicates the transform of the matrix of reduced variables (discussed below). The author uses $\boldsymbol{C}$ to indicate the correlation matrix, for clarity, however, other authors have used $\boldsymbol{C}$ for the covariance matrix, and $\boldsymbol{R}$ for the correlation matrix (e.g. Low 2008).

2.6 Limit State Surface

The limit state surface, or failure surface, can be defined as the unfactored difference between the resistance and load of a system, as shown in Figure 2. The definition used here is given my Equation 7.

$$M = R - L \quad [7]$$

Where R is the resistance and L is the load. The definition of the limit state surface can have many forms. The most convenient form for the FORM is the one defined by Equation 7 as designers are interested in how far the mean of the safety margin (i.e. when $R < L$, or $M < 0$) is from the failure point (Fenton and Griffiths 2008), or the design point as shown in Figure 2.

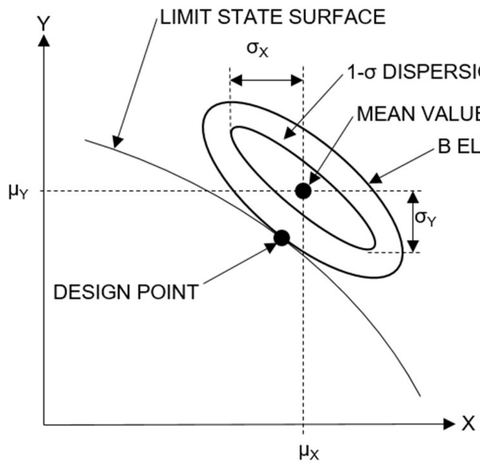

Figure 2. Illustration of FORM in relation to the limit state surface and reliability index (after Low 2008)

The definition of M per Equation 6 allows the reliability index, which is discussed below, to be minimized by setting searching for the value of the vector of reduced variables such that Equation 6 is at it minimum value, and Equation 7 is zero.

The limit state function and its relationship to the reliability index, which is discussed below, is illustrated in Figure 2.

In Figure 2, the mean value point represents the state calculated when only the mean values of the independent random variables, dependent variables, and deterministic variables are considered. The design point represents the intersection of the limit state surface and the minimized reliability index when the matrix of reduced variables is set to some value.

For the mean values used by the author, when the limit state function is set to zero, and the reliability index is minimized, the probabilistic analysis using FORM results in a probability of failure of 50%. Considering Figure 2, this would mean that the design point and the mean point would be co-located on the limit state surface, M and 50% of the ellipse representing the reliability index would be above M and the other 50% would be located below M. Considering the physical meaning, this would mean that one increment of standard deviation in one direction would have the same but opposite effect as one increment of standard deviation in the opposite direction. With 50% of the ellipse representing β on either side of M, the probability of failure would be 50%.

2.7 Reliability Index

The reliability index is used to measure the distance to the limit state surface. The probability of the distance to M being less than zero is defined by Equation 8.

$$p_f = 1 - \Phi(\beta) \quad [8]$$

Where p_f is the probability of failure and Φ is the normal standard distribution. Given that the author deployed FORM is a spreadsheet program, the normal standard distribution is provided as a built in function and hence, p_f is easily calculated.

3.2 Matrix of Reduced Variables

FORM in essence measures the number of standard deviations a point is from the failure surface. The vector of reduced variables, $\boldsymbol{X_r}$ in Equation 5 is defined by Equation 9 and is illustrated in Figure 2.

$$\boldsymbol{X_r} = \frac{x - E[\boldsymbol{X}]}{\sigma_X} \quad [9]$$

The solution to the FORM equation presented as Equation 5 is the minimum value of β. In this case, the minimum value of β is found by finding a combination of reduced variables such that β is minimized. The minimum value of β can be found by using the add-in Solver function in Excel (Foreman 2014). One only needs to set β to be minimized with a constraint of $M = 0$.

3.3 Correlation

Correlation provides a method to determine how changing the value of one variable by a number of standard deviations affects another variable it is correlated with. In commercially available software, like RocPlane (Rocscience 2017) and Slope/W (GEO-SLOPE 2012), correlation is ignored when carrying out a sensitivity analysis.

Figure 3 provides the sensitivity analysis for the limit equilibrium analysis carried out in RocPlane using the mean values presented in Tables 1, 2, and 3. The minimum variable, the one represented by the line below all others (H_f in the case of Figure 3 for negative percent change from the mean and c for positive percent change from the mean). The minimum variable controls the stability of the system, governing the outcome of the assessment.

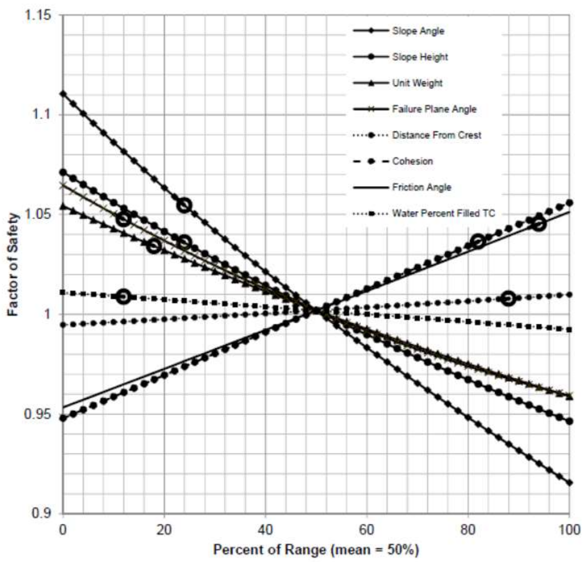

Figure 3. Graph of sensitivity analysis carried out in RocPlane (Rocscience 2017)

The correlation matrix, $\boldsymbol{C}$, in Equation 5 is given by Equation 10.

$$\boldsymbol{C} = \begin{matrix} \rho_{ii} = 1 & \cdots & \rho_{ij} \\ \vdots & \ddots & \vdots \\ \rho_{ji} & \cdots & \rho_{jj} = 1 \end{matrix} \qquad [10]$$

The matrix for **C** in Equation 10 will be as wide and tall as there are independent random variables and dependent variables, n, where i is the variable in the first column and first row, and j is the variable in the last column and last row. Since the coefficient of correlation for a single variable is one, then the coefficients along the diagonal of the matrix will be equal to one.

The problem with only sampling one variable at a time can be shown in Figure 3. The plots in Figure 3 show the relationship between the variables and the factor of safety (another name for the limit state surface). The plot that controls the value of the factor of safety lies below the remaining plots. Figure 3 shows that for values less than 50% of the range of a given mean value, cohesion controls the value of the factor safety (as discussed above), but for values greater than 50% the slope height does. If all the variables were randomly sampled from their respective probability functions, it would be possible for the probability of failure to be less than 100%, as it is with the FORM method, as indicated by the black circles in Figure 3. In other words, some combination of values above the horizontal for a factor of safety equal to one can theoretically occur. Statistically speaking, if half the total length of lines lie above one, and half below, the probability of failure would be 50%.

3.4 Probability of Meeting Acceptance Criteria

As discussed above the probability of failure can be defined by Equation 7. The probability of failure is not unique in the sense that failure can be defined in a number of different ways, such as the ultimate limit state, serviceability limit state, or as falling below a given factor of safety, i.e. the acceptance criteria. The example used by the author set the factor of safety for a planar-type failure as unity. From this the probability of failure can be assessed using FORM by minimizing the reliability and setting the limit state function to zero by changing the matrix of reduced variables to the required set of values.

Another use of FORM is to assess how far from the mean that data can be taken to reach an acceptable level of risk. In essence, what is the level of confidence the designer can have in the data to produce an acceptable outcome.

4 DISCUSSION

4.1 Data Collection

The author's experience preparing this paper has reinforced upon him the need for high quality geological data for even the simplest of deterministic analyses. The use of probabilistic methods requires the utmost care in scrutinizing the data for quality. Outliers can easily skew a distribution towards erroneous means and variances, resulting in invalid solutions to problems.

Today's technology (ground and air based photogrammetry and LiDAR, InSAR, etc.) provide designers with an opportunity to collect vast amounts of data that when used appropriately, can provide a wealth of information. However, ground truthing and validation against one or more other data collection techniques should not be abandoned. It is very likely that a line or window survey of a rock mass with a geological compass and one's bare hands will never truly be replaced by technology. That being said, manual, or "analog" methods of data collection should be revised to match that of digital techniques so that manual data collection can be used to back-check digital mapping. For instance, persistence has typically been mapped as the longest surface expression of a plane regardless of whether it is parallel to the slope face, which would equate to the "out-of-plane" dimension in a 2D analysis, or perpendicular to the slope face, which would be the plane of sliding. The author has had a few, and at times heated debates, over which direction is more important. In reality both are, as each gives a unique measure of the slope and each can be used to back-check the calculation of dip and dip direction calculated by digital mapping software (Haneberg et al. 2006).

Consistency is also a key element of any data collection program. If not simply to make corrections to errors in the method of collection, but to make sure that post-processing calculations to not lead to errors in the designs engineers produce. For instance, all measurements during a geological mapping program should be measured horizontally and vertically, or parallel to the feature being mapped. Switching back and forth will result in errors when calculations, such as for true spacing which would be used to assess the potential location of tension cracks.

4.2 Statistical Distribution of Data

It is imperative that the data be fitted to the appropriate distribution before proceeding with any probabilistic analyses. Basic assumptions regarding the mean and standard deviation based on the notion that any data will fit a given distribution without fully assessing that fact, could lead to erroneous results.

As described above, the synthetic data for the measured independent variables was set at random log-normally distributed data, and then the normal mean, variance, standard deviation, and arithmetic moments were calculated. The depth of water in the tension crack, and acting on the failure plane, a truncated exponential distribution was used with the mean set at 50% of the mean depth of the tension crack. A truncated exponential distribution was also used to limit the height of the failure block in the range of 60 m to 100 m.

Data that the author is most familiar with tends to fit a log-normal distribution, hence that distribution was chosen for the synthetic data use in this study. That is not to say that other distributions would not be more suitable for natural data sets. For instance, the poles plotted on a stereonet are assumed to fit a Fisher distribution (Diederichs 1990). This would need to be considered in probabilistic assessments where the dip and dip direction are both used, however, in the case of planar failure, only the dip of the planar sliding surface need be considered, and should be fitted to a discrete univariate distribution.

The truncated exponential function was used for the height of the failure block and height of water in the tension crack. The height of water will never be greater than the depth of the tension crack and never less than zero. The truncated exponential distribution is ideal for bounding the values of z_w to this range. The same can be said for the height of a failure block if it is known to have a fixed range of heights between zero and a non-negative integer or two non-zero and non-negative integers.

4.3 Complexity of Analysis

Examples of the FORM being used to assess the probability of failure for rock slopes have tended to focus only two or three independent random variables (i.e. Low 2008). In this study, the correlation matrix is developed for all but three of the independent random variables that could otherwise be included (as noted above JRC, JCS, and φ_r were used as deterministic variables). As far as the author can determine, this is the most complex assessment of the probability of failure of a planar-type failure mechanism in a rock slope using the FORM that has been carried out to date.

The correlation matrix for all the independent variables considered, required the computation of 190 equations of covariance and subsequently correlation coefficient. This resulted in the production of two 20 by 20 matrices. However, as the author has pointed out, the covariance of a variable with itself is one, solutions to 20 equations of covariance are known. Additionally, as independent variables are not correlated, by definition, a further nine solutions are known. More solutions can easily be known by visually assessing the equations for each dependent variable to determine if correlation exists between variables.

4.4 Rock Support

The inclusion of rock support to the analyses would require defining the tensile and/or shear capacity of the support and the angle at which it is installed. Including rock support in the analyses will result in the mean point being moved further away from the design point. The matrix of reduced variables will then be recalculated resulting in a lower probability of failure since β will be larger.

5 CONCLUSIONS

This paper summarizes the implementation of FORM in a spreadsheet to assess the probability of failure of a planar-type failure in a rock mass. The problem is illustrated by defining the independent random variables and the dependent variables. Although all variables can be either independent or dependent, the author chose to keep the shear strength variables as deterministic to control the outcome of the solution to illustrate the power of FORM.

Although FORM was shown to be easy to implement in a spreadsheet, the number of variables that can be independent and dependent can make the execution of FORM complex. However, once the problem is defined and implanted, it can be replicated again and again using the same spreadsheet.

The author intends to implement FORM for other uses in the future to aid in the assessment of risk. As long as there is a closed-form solution for a problem, and a failure surface can be defined, FORM can be used to assess the probability of failure.

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