Shear Strength Assessment of Run-of-Mine Waste Rock Used as Support Fill at the Aitik Mine Through Empirical Approaches

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Abstract

The shear strength of coarse rockfill materials is typically evaluated through large-scale laboratory testing; however, such testing is often impractical due to cost and equipment limitations. This study presents a geotechnical characterization of run-of-mine waste rock used as support fill at the Aitik Tailings Management Facility (TMF) in Sweden, with the objective of estimating shear strength parameters using empirical methods.

Thirteen sampling locations were investigated to capture material variability. A combined approach using digital image analysis (>75 mm) and sieve analysis (<75 mm) was adopted to derive a representative particle size distribution (PSD). Bulk unit weight was obtained using LiDAR-derived volumes and haul truck mass data, while point load testing was used to estimate particle strength.

Four empirical methods (Leps, Barton and Kjærnsli, Douglas, and Leps 2.0) were applied using the derived parameters. All empirical methods produced consistent nonlinear shear strength envelopes, with predictions clustering within a narrow range, indicating good agreement between the methods and supporting their combined use as a reasonable basis for engineering assessment, in the absence of site-specific large-scale laboratory shear strength testing.

Introduction

Blasted and crushed waste rock generated in mining operations is typically stored in waste dumps or stockpiles, or used as construction material in facilities such as tailings management facilities (TMFs). These materials are inherently heterogeneous, with particle sizes ranging from fines to meter-scale boulders.

Conventional sieve-based methods are impractical for large-scale materials due to time, handling constraints, and the presence of oversized particles. Additionally, laboratory testing to determine shear

strength is often limited by specimen size constraints, requiring scaling techniques that may alter the material behavior. As a result, empirical methods provide practical alternatives to large-scale testing but rely heavily on representative input parameters such as PSD, density, and particle strength.

This study presents a workflow for estimating the shear strength of waste rock using multiple empirical methods, based on field and laboratory characterization of material used at the Aitik TMF.

Methodology and Main Inputs

This study focuses on the waste rock support fill for the tailings management facility (TMF) at the Aitik copper mine in northern Sweden, within the Arctic region. The facility covers roughly 16 km² and has operated continuously since 1968. Run-of-mine waste rock at the TMF was characterized using field sampling, PSD, density estimation, particle strength testing, and empirical methods. Figure 1 presents a plan view of the Aitik TMF, highlighting the dams included in the analysis.



Figure 1: Aitik TMF plan view

Particle Size Distribution and Shape

To characterize the full particle size distribution (PSD) of the waste rock, a combined approach was adopted using digital image analysis for coarse particles (>75 mm) and conventional sieve analysis for finer fractions (<75 mm). Due to the widely graded nature of the material, image-based results may underestimate finer fractions; therefore, a procedure is proposed to obtain a more representative PSD.

Sampling Program

Thirteen (13) sampling locations were selected across the TMF to capture the spatial variability of the waste rock material. Sampling locations were selected by slowly driving along the entire length of each buttress or stockpile to identify representative areas, including estimating visible fines content, while avoiding slope toes where coarser fractions tend to accumulate.

The locations where photographs were taken and samples collected across all dams. At each location, a disturbed sample of approximately 50 kg was collected, and a high-resolution photograph with a width exceeding 2 m was taken.

Sieving

Particle size testing was conducted on 13 waste rock support fill samples listed in the Sampling Program section. The particle size distributions were determined with samples scalped at 75 mm, and sieving was carried out at the Klohn Crippen Berger (KCB) site laboratory.

Digital Image Analysis

The software WipFrag version 4 (WipWare, 2025) was used to delineate particles from photographs. An example is shown in Figure 2, where the original photograph of waste rock at Dam C-D (Sample 1) is presented alongside the corresponding image analysis mesh and digital image representation.

It should be noted that areas appearing to contain a high proportion of finer material are sometimes interpreted in the photographs as collections of gravel-sized particles, as illustrated in Figure 2 (the red circles highlight areas of finer particles incorrectly identified as individual particles), suggesting that the results might be underestimating the finer fraction in widely graded soils. To account for this limitation, finer fractions can be assessed using the built-in calibration modules in the software, supplemented with conventional sieving or multi-scale photographic analysis. In this study, conventional sieving was used to supplement the image analysis.

Using sieving results, the proportion passing the 75 mm sieve obtained from the uncalibrated image analysis was compared with the corresponding sieving results, as shown in Figure 3 for two PSD samples from Dam C-D, confirming the underestimation of finer particles. Consequently, sieve analysis results were adopted for particle sizes smaller than 75 mm, while WipFrag-derived results were used for the coarser fraction above 75 mm.

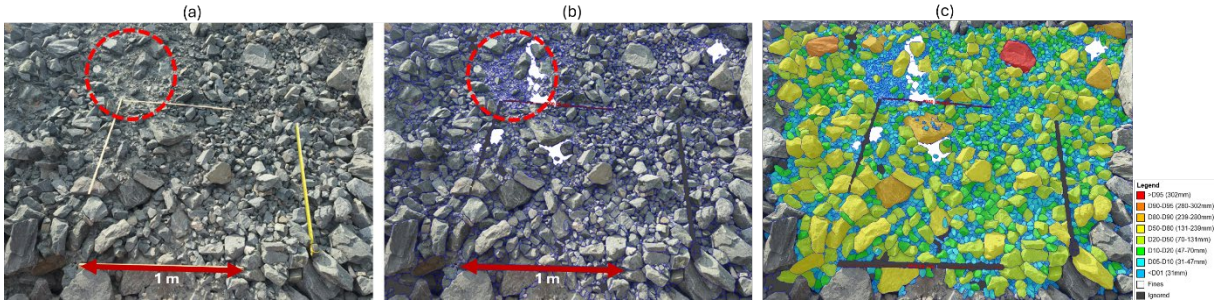


Figure 2: (a) Original photography for waste rock at Dam C-D Sample 1; (b) Image analysis results mesh; (c) Corresponding digital image representation

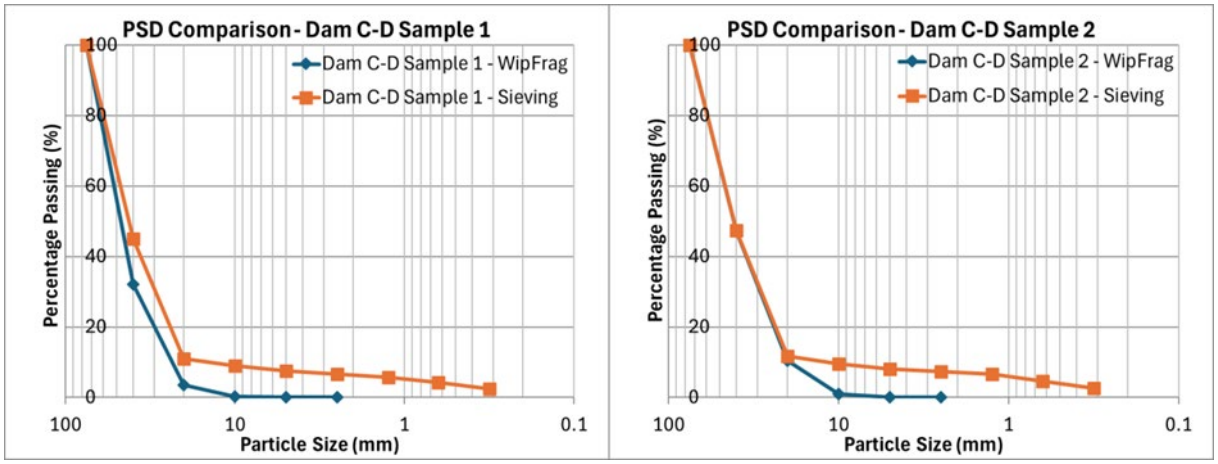


Figure 3: Comparison of fraction passing 75 mm from uncalibrated digital image analysis with sieving results

Proposed Procedure

Digital image results obtained without calibration represent only the particle size distribution (PSD) of particles visible at the exposed surface. Because particles hidden beneath the surface are not accounted for, this underestimates the finer fractions and biases the resulting PSD toward coarser particle sizes in widely graded materials. To address this limitation, an approximation procedure was developed to estimate an equivalent volumetric PSD.

Particle counts for each size fraction were extracted from the particle size histogram and normalized to a fixed reference depth of 1 m, assuming continuity of particle size distribution behind the visible surface. Particle volumes were then calculated assuming spherical geometry, and cumulative passing percentages were derived following standard procedures. Assuming uniform particle density across all size fractions, the resulting volumetric PSD was taken as equivalent to a mass-based distribution and combined with the sieve analysis data to produce a unified PSD. This procedure should be regarded as a practical engineering tool for characterizing waste rock where direct large-scale PSD measurement is impractical.

Combination of Sieving with Digital Image Analysis Results

A combined methodology was adopted to determine the full-range particle size distribution (PSD), using the digital image analysis results for particles larger than 75 mm and laboratory sieve analysis for particles smaller than 75 mm. The resultant full PSDs (after merging) are presented in Figure 4.

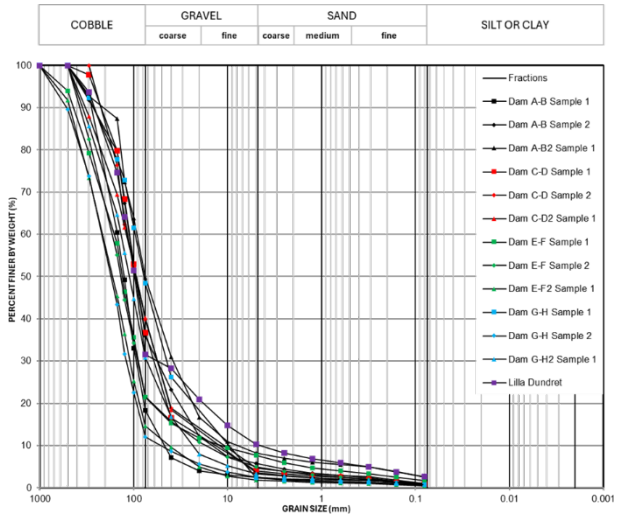


Figure 4: Particle size distributions using combined digital image analysis and sieving results

Variability of Grading

As shown in Figure 4 above, all particle size distribution (PSD) curves fall within a defined gradation band, indicating a consistent overall grading envelope. However, the dispersion of the individual curves highlights the inherent variability of the material. This variability is expected, as the waste rock is essentially unprocessed and unsorted.

Unit Weight Determination

Bulk unit weight was determined using a combination of load-cell measurements on haul trucks to obtain mass and Light Detection and Ranging (LiDAR)-derived volumes obtained from drone surveys.

The bulk unit weight was calculated by dividing total mass by the corresponding placed volume. Estimates of the bulk unit weight of the rockfill placed during each construction interval were calculated by dividing the total tonnage recorded for each interval by the corresponding measured volume.

From the compiled values, an average bulk unit weight of 21.1 kN/m³ was obtained. Assuming an average moisture content of 5%, this corresponds to a dry unit weight of approximately 20.1 kN/m³. It should be noted that these unit weights are representative of material placed near the ground surface. As the buttress height increases, the unit weight is expected to rise due to increased compaction under self-weight. Consequently, the reported values should be considered as lower-bound estimates.

Particle Strength

Particle strength was determined through point-load testing on rock fragments between 20 mm and 100 mm. A total of 35 rock samples were tested. Of these, 14 exhibited planar failure modes and were therefore considered suitable for further analysis.

The point load index was converted to unconfined compressive strength (UCS) using a preliminary relationship between the Unconfined Compression Strength (UCS) and the Corrected Point Load Strength ($I_{s(50)}$) equal to UCS = 24$I_{s(50)}$. The calculated UCS values for the valid samples reflected the variability and inherent heterogeneity of the rockfill material. The average UCS derived from the dataset is 98.5 MPa.

Summary of Properties of the Aitik Waste Rock Support Fill

The average PSD indicates a median particle size (D_{50}) of 111.2 mm, with a coefficient of uniformity (C_u) of about 5.3 and a coefficient of curvature (C_c) of roughly 1.6, suggesting a well-graded material with a broad range of particle sizes. The key index properties of the Aitik waste rock support fill are summarized in Table 1.

Table 1: Properties of the Aitik Waste Rock Support Fill Based on the Average PSD

Dry / Bulk Unit Weight (kN/m³)	Uniaxial Compressive Strength, (MPa)	D_{10} (mm)	D_{15} (mm)	D_{30} (mm)	D_{50} (mm)	D_{60} (mm)	Coefficient of Uniformity, C_u (-)	Coefficient of Curvature, C_c (-)
20.1 / 21.1	98.5	25	34.2	72.7	111.2	133.5	5.3	1.6

Where: D_{10}, D_{15}, D_{30}, D_{50}, and D_{60} are the particle sizes corresponding to 10%, 15%, 30%, 50%, and 60% passing, respectively.

Shear Strength Estimation

Shear strength was estimated using four empirical approaches: Leps (1970), Barton and Kjærnsli (1981), Douglas (2002), and the Leps 2.0 framework (Cylwik et al., 2025).

Mathematical Relationships to Estimate Shear Strength

Table 2 summarizes the shear strength methods considered in this study alongside their corresponding mathematical relationships, providing a consistent basis for comparison between approaches.

Table 2: Shear Strength Methods and Mathematical Relationships

Author	Mathematical Relationship	Parameters
Leps (1970)	$\tau = \sigma'_n \tan(\phi'_0 - 6.6614\log(\sigma'_n))$	ϕ'_0: 55.3, 60.6, and 65.5 for lower, average, and upper bounds at 1kPa; σ'_n:Effective normal stress (kPa)
Barton and Kjærnsli (1981)	$\tau = \sigma'_n \tan\left(R\log\left(\frac{S}{\sigma'_n}\right) + \phi_b\right)$	R: Equivalent roughness; S: Equivalent particle strength; ϕ_b : Basic effective friction angle
Douglas (2002)	$\tau = \sigma'_n \tan(a + b(\sigma'_n)^c)$	ANG: Angularity rating; a, b: Coefficients; c: Exponent
Leps 2.0 (Cylwik et al., 2025)	$\tau = \tan(\phi'_1)\,(10P_a)^{1-m}(\sigma'_n)^m$	ϕ'_1: Secant friction angle; P_a: Atmospheric pressure (kPa)

Where: τ is the shear stress, and σ'_n is the normal effective stress.

Definition of Leps Parameters

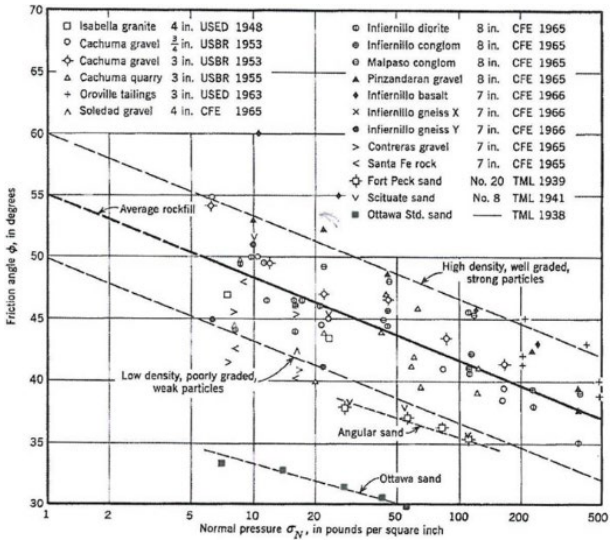


Figure 5: Shearing strength of rockfill from large triaxial tests (Leps, 1970)

Figure 5 illustrates the shear strength characteristics of rockfill materials based on large-scale triaxial test results reported by Leps (1970).

It should be noted that, apart from particle strength, Leps (1970) does not provide quantitative definitions for the qualitative descriptors used in the chart. In particular, terms such as “high” or “low” density and “well-graded” or “poorly graded” materials are not explicitly defined. To enable application of this framework to the Aitik waste rock support fill, appropriate interpretations of these descriptors are

required. Accordingly, definitions tailored to the characteristics of the Aitik material are proposed in Table 3.

Table 3: Definitions of Parameters Used in the Leps Curves for Use at Aitik

Parameter	Proposed Definition	Proposed Method to Derive Parameter
Particle size distribution	Standard Practice for Classification of Soils for Engineering Purposes (Unified Soil Classification System) (ASTM D2487-17e1 (2017))	Digital image analysis merged with traditional sieving analysis
Bulk density	Table 4	Measurement of tonnages and volumes of trucks carrying support fill waste rock
Particle strength	Leps (1970)	Point Load Testing (PLT)

From the shape parameters, the terms "well graded" and "poorly graded" can be defined using the Unified Soil Classification System (USCS) criteria (ASTM, 2017). Soil is considered well graded when $Cu>4$ and $1<Cc<3$, whereas any soil that does not satisfy these criteria is classified as poorly graded.

The Leps curves describe material density using qualitative terms such as "high density" and "low density." Although the original work by Leps refers to dry density, bulk density values have been adopted in this assessment (see Tables 2 and 3) to ensure consistency with the available project data.

Guidance from the literature provides context for interpreting these density ranges. ICOLD (2011), in relation to the construction of concrete-faced rockfill dams (CFRDs), indicates that compacted rockfill densities typically range from approximately 1,900 kg/m³ at the base of poorly graded layers to around 2250 kg/m³ at the top of well-graded layers. This variation reflects the increase in compaction energy with elevation during placement. Similarly, Valenzuela et al. (2008), in the context of waste rock dumps, report a broader range of rockfill densities, between approximately 1,800 kg/m³ and 2,300 kg/m³.

Based on these references, a density classification system has been developed (Table 4) to provide a structured framework for describing the waste rock support fill.

Table 4: Proposed Range of Bulk Densities for the Aitik Waste Rock Support Fill

Density (kg/m³)	Unit Weight (kN/m³)	Descriptive Term	Equivalence in Leps (1970) Method
1,800–1,900	17.7–18.6	Very loose	Low density
1,900–2,000	18.6–19.6	Loose	Low density
2,000–2,100	19.6–20.6	Medium dense	Medium density (not explicitly mentioned)
2,100–2,200	20.6–21.6	Dense	High density
2,200–2,300	21.6–22.6	Very dense	High density

The classification of rock particles proposed by Leps (1970) defines three strength categories (weak, average, and strong) based on unconfined compressive strength ranges. Although the original ranges are presented in pounds per square inch (psi), they are expressed here in SI units for consistency: weak particles correspond to strengths between 3.5 and 17.2 MPa, average particles range from 17.2 to 68.9 MPa, and strong particles fall between 68.9 and 206.9 MPa.

Assessment of Site-Specific Parameters—Leps (1970)

Considering the characteristics of the waste rock support fill described in the preceding sections, Table 5 presents the estimates together with their corresponding qualitative assessment.

Table 5: Assessment of Site-Specific Parameters—Leps (1970)

Parameter	Current Value Estimate	Qualitative Equivalent	Outcome
Particle size distribution (for the average curve)	C_u = 5.3 C_c = 1.6	Well graded	Upper-bound parameters indicated; average Leps curve adopted to account for material variability
Bulk/dry density	ρ_{bulk} / ρ_{dry}= 2,150 / 2,050 kg/m^3	High density (Dense, based on Table 4)	
Particle strength	UCS = 98.5 MPa	Strong	

Assessment of Site-Specific Parameters—Barton and Kjærnsli (1981)

Barton and Kjærnsli (1981) identified a close correspondence between the non-linear shear strength behavior of rock joints and that of rockfill. Based on a comparative analysis of roughness, strength, and friction angle characteristics of rockfill and rock joints, Barton and Kjærnsli (1981) developed a relationship for defining the friction angle of rockfill. The parameters used in this approach are summarized in Table 6.

Table 6: Assessment of Site-Specific Parameters—Barton and Kjærnsli (1981)

Parameter	Input	Estimated Value
Equivalent roughness (R)	Type: Quarried rock, angular; G_s=2.7; ρ_d=2,050 kg/m^3; n=0.24 (24%) Figure 6(a)	R=8
Equivalent strength (S)	D_{50}=111.2 mm; UCS=σ_c =98.5 MPa Figure 6(b)	S/σ_c=0.25; S=24.6 MPa
Basic friction angle (ϕ_b)	For intact rock: ϕ_b between 25 and 35°	ϕ_b=30°

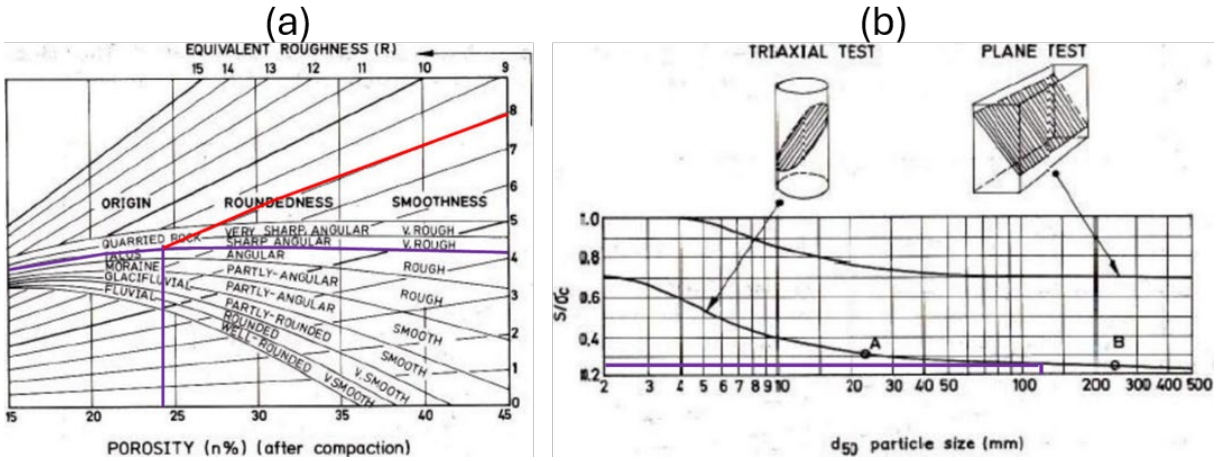


Figure 6: (a) Equivalent roughness (R) and (b) equivalent strength (S) estimation

Assessment of Site-Specific Parameters—Douglas (2002)

A statistical analysis was undertaken to evaluate the influence of various factors governing the shear strength behavior of rockfill materials. Table 7 summarizes the assessment of site-specific parameters using Douglas (2002).

Table 7: Assessment of Site-Specific Parameters—Douglas (2002)

Expressions	Parameters Definition	Estimated Value
a=36.43-0.267ANG-0.172FC+0.756(C_c-2) +0.0459(UCS-150)	ANG=Angularity rating-5 (for $ANG > 5.5$); otherwise, ANG=0; ANG goes from 1 (rounded) to 8 (angular); FC: Fines content (< 0.075 mm); C_c: Coefficient of curvature; UCS: Uniaxial Compressive Strength (MPa)	Using: ANG=2 (angular particles with angularity ratio equal to 7); FC=1%; C_c=1.6; UCS= 98.5 MPa
b=69.51+10.27ANG+0.549FC-5.105(C_c-2) -0.408(UCS-150)-0.408		
c=-0.3974		The following values are obtained: a=33.1; b=113.3; c=-0.3974

Because the Douglas (2002) method produces high friction angle values at low effective normal stresses, a cap of 60° was applied.

Assessment of Site-Specific Parameters—Leps 2.0 (Cylwik et al., 2025)

The Leps 2.0 framework introduces a more structured nonlinear formulation based on anchor stress conditions and curvature of the shear envelope. This method expresses shear strength using a nonlinear power-law formulation presented in Table 2.

For the estimation of the main parameters using this methodology, the studies by Cylwik et al. (2025), as well as the user guideline outlining the step-by-step application of the method presented by Arrieta (2025), have been reviewed.

Despite the absence of laboratory shear strength data, the Leps 2.0 method (Cylwik et al., 2025) can be applied. However, its use without such data is recommended only for preliminary or conceptual evaluations.

To verify its applicability, it is necessary to confirm that the gravel content exceeds 40% and that the fines content is less than 25%. In addition, the median particle size (D_{50}) must be greater than 2 mm.

Based on the data previously presented (Figure 4 and Table 1 for the average PSD), these criteria are satisfied. Therefore, the Aitik waste rock support fill is considered suitable for analysis using this methodology.

Curvature Coefficient

When laboratory testing has not been carried out, the parameter m is typically assumed to be 0.9, which is a representative value for rockfill materials in the absence of site-specific laboratory data. Given this, a value of 0.9 has been adopted for the coefficient of curvature for this assessment.

Secant Friction Angle

Two alternative approaches can be adopted to estimate the secant friction angle in the absence of laboratory test data. The first applies when field-measured void ratio values are not available. In this case, the secant friction angle is estimated using a double-entry table, where rockfill quality (classified as good, moderate, or low) is considered along one axis and the degree of compaction along the other. Typical void ratio values are assumed for each compaction state, with dense corresponding to a void ratio of 0.3, medium-dense to 0.45, and loose to 0.6. The second approach is used when the void ratio is available or can be reasonably estimated. For this approach, where the void ratio can be estimated, the following expression may be used:

$$\phi'_1 = \phi'_{e=0} - 20e \quad (1)$$

Where:

ϕ'_1: Secant friction angle; $\phi'_{e=0}$: 53°, 50°, and 47° for good, moderate, and low rockfill quality, respectively;
e: Void ratio

The bedrock at Aitik is predominantly composed of high-grade metamorphic rocks, mainly granite gneiss of varying composition with subordinate intrusive granite and minor meta-arenite. The Aitik waste rock can be classified as moderate-quality material based on its lithology and origin. Accordingly, a value of $\phi'_{e=0} = 50°$ has been adopted, with a void ratio of 0.316 (corresponding to an assumed porosity of 0.24). Therefore, using Equation 1, the secant friction angle is estimated as 43.7°.

Shear Strength Results and Comparison

Figure 7 presents a comparative analysis of shear strength envelopes derived from the different empirical models using the site-specific parameters suggested in this section. Mathematical expressions for all the methods, except for Leps, are included in the figure.

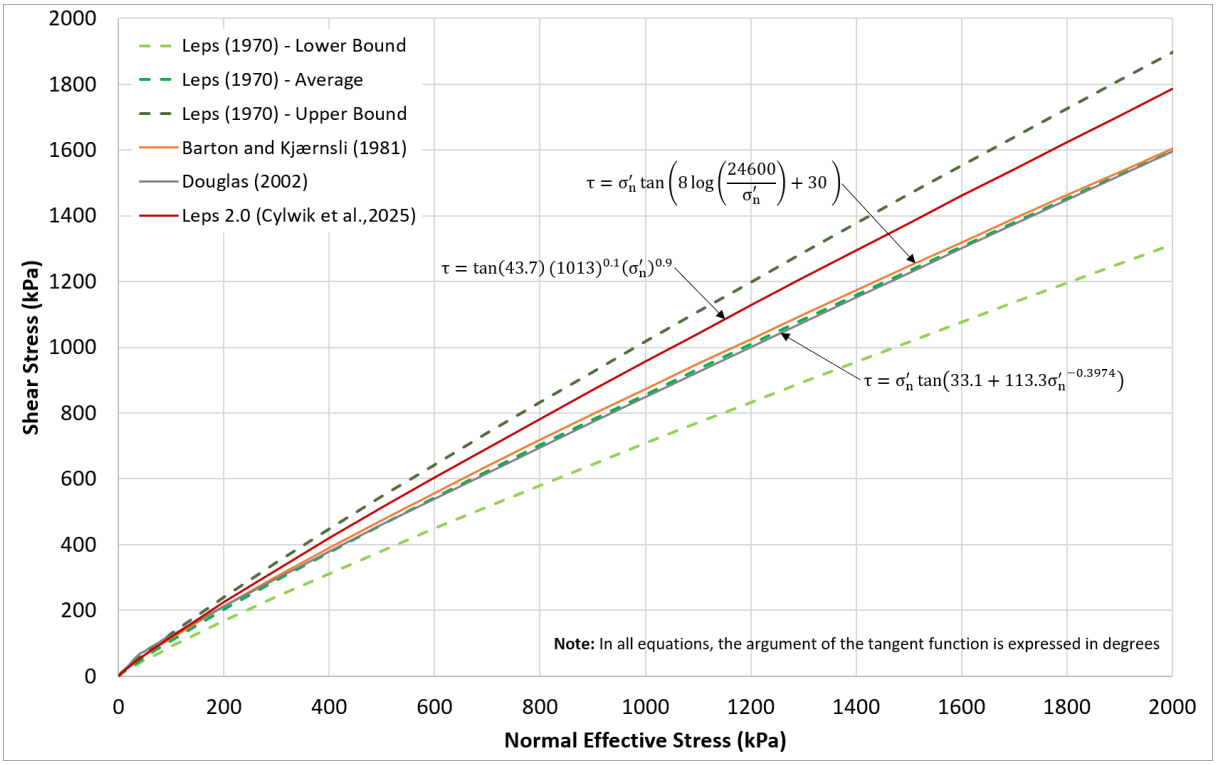


Figure 7: Comparison between shear strength envelopes

Comparison of Methods

The shear strength envelopes derived from the different empirical methods show consistent nonlinear behavior over the full range of normal effective stresses considered.

The predictions from the Barton and Kjærnsli (1981), Douglas (2002), and average Leps (1970) methods are closely aligned, with only minor differences across the stress range. The Leps upper bound and Leps 2.0 methods define a slightly higher envelope, while the Leps lower bound provides a conservative estimate. For the range of normal effective stresses presented, the variation between methods remains within a relatively narrow band (approximately ±20% around the central trend), indicating good agreement.

Conclusion

This study presents a practical methodology for estimating the shear strength of waste rock in the absence of large-scale shear strength laboratory testing. The methodology combines digital image analysis, sieve

analysis, unit weight determination, and particle strength testing to derive the input parameters required for empirical shear strength assessment.

The approach was applied to the Aitik waste rock support fill, and four empirical methods, namely Leps, Barton and Kjærnsli, Douglas, and Leps 2.0, were subsequently used to estimate shear strength. Although the results are subject to uncertainty associated with material variability, the PSD characterization procedure, and the use of empirical correlations, the four methods produced consistent nonlinear shear strength envelopes clustering within a narrow range. This agreement between methods supports their combined use as a reasonable basis for engineering assessment of the waste rock support fill, in the absence of site-specific large-scale laboratory shear testing.

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